



Proceedings of the Eurosensors XXIII conference

Experimental validation of hardening and softening resonances in a clamped-clamped beam MEMS resonator

R.M.C. Mestrom^{*a}, R.H.B. Fey^a, K.L. Phan^b, H. Nijmeijer^a^aDepartment of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands^bNXP-TSMC Research Center, NXP Semiconductors, P.O. Box WAG 02, 5656 AE Eindhoven, The Netherlands

Abstract

By means of a combined analytical-numerical and experimental approach, the nonlinear dynamic behavior of a clamped-clamped beam MEMS resonator has been investigated. A good quantitative correspondence between simulations and experiments has been obtained. First-principles based multiphysics modeling is applied to derive a reduced-order model of the resonator. The model includes nonlinear geometric and electrostatic effects as well as thermoelastic damping and anchor loss. Both simulations and experiments show hardening and softening nonlinear dynamic behavior depending on the excitation parameters. The model captures the observed nonlinear behavior and allows for design optimization with respect to nonlinear effects.

Keywords: Multiphysics modeling, MEMS resonator, nonlinear dynamic behavior, experimental validation

1. Introduction

Single-crystal silicon MEMS resonators provide an interesting alternative for quartz crystals in time reference oscillators [1]. Their small size allows for on-chip integrability but makes them more susceptible to nonlinear effects since they have to be driven into nonlinear regimes in order to store enough energy [2]. Different nonlinear effects may be dominant in the dynamic behavior, depending on the resonator design and the operating conditions. Since, the presence of nonlinearities is relevant for oscillator performance, their analysis has to be incorporated in resonator design optimization.

In this paper, a predictive modeling approach is proposed that provides designers with a powerful characterization and optimization tool for nonlinear effects in MEMS resonators. The work presented here extends lumped modeling efforts for a similar resonator [3,4] towards a generally applicable first-principles based multiphysics modeling approach that allows for fast and accurate simulation of the dynamic behavior of MEMS resonators. The modeling approach is illustrated for a clamped-clamped beam MEMS resonator that features various nonlinear effects. Furthermore, the derived model is validated experimentally. In this way, confidence is gained in the model, which can be used for prediction of the nonlinear behavior and for optimization of the resonator design.

* Corresponding author. Tel.: +31 40 2472811; fax: +31 40 2461418.

E-mail address: r.m.c.mestrom@tue.nl.

The outline of the paper is as follows. First, in Section 2, the clamped-clamped beam MEMS resonator will be described. In Section 3, the multiphysics modeling approach for the resonator will be presented. Results for both simulations and experiments will be presented in Section 4. Finally, some conclusions will be drawn in Section 5.

2. Clamped-Clamped beam MEMS resonator

Consider an electrostatically actuated clamped-clamped beam MEMS resonator. A microscope image of the resonator is depicted in Fig. 1(a) and a schematic layout is shown in Fig. 1(b). The resonator beam has a length of $l = 47.75 \mu\text{m}$, a width of $h = 4 \mu\text{m}$ and a thickness of $b = 1.4 \mu\text{m}$ (out-of-plane). The electrode gaps are $d_1 = d_2 = 342 \text{ nm}$. The resonator is actuated by a dc voltage V_{dc} and an ac voltage V_{ac} . The output of the resonator V_{out} results from capacitive detection. Six aluminum bond pads can be distinguished in Fig. 1(a). These are designed to fit the probes that are used in the measurements. The outer four bond pads are connected to ground and the middle two bond pads are used for actuation and measurement purposes. In the experimental setup, resonator oscillations are measured by sweeping the ac frequency f up and down around the natural frequency of the resonator (which is around 13 MHz).

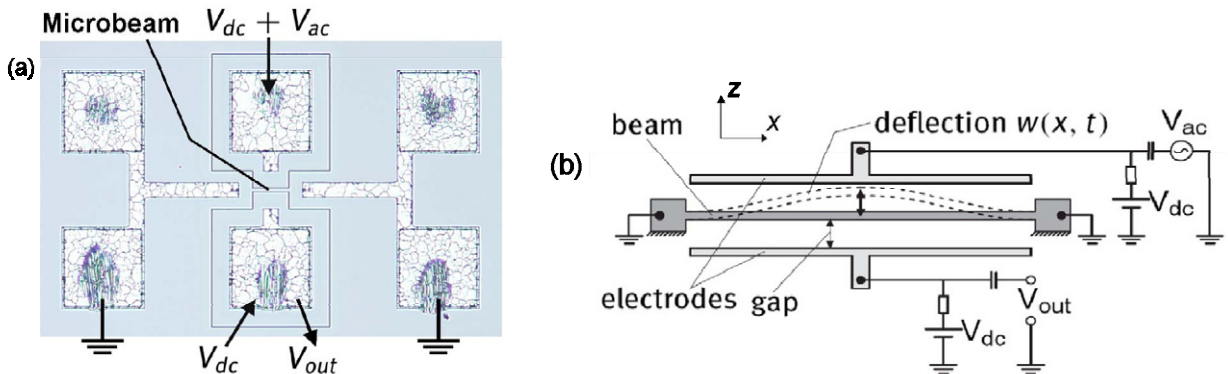


Fig. 1. Clamped-clamped beam MEMS resonator: (a) microscope image; (b) schematic layout.

3. Modelling approach

The multiphysics model that describes the dynamics of the resonator (see Fig. 1(b)) is based on Timoshenko beam theory and describes both the transverse deformation $w(x, t)$ and shear deformation $\varphi(x, t)$ of the beam, using the following partial differential equations:

$$\rho A \frac{\partial^2 w}{\partial t^2} - k_s GA \left(\frac{\partial \varphi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) - \left[\frac{EA}{2l} \int_0^l \left(\frac{\partial w}{\partial x} \right)^2 dx - \frac{1}{l} \int_0^l N_T dx \right] \frac{\partial^2 w}{\partial x^2} = q_e, \quad (1)$$

$$\rho I \frac{\partial^2 \varphi}{\partial t^2} - EI \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial M_T}{\partial x} + k_s GA \left(\varphi + \frac{\partial w}{\partial x} \right) = 0. \quad (2)$$

Here, ρ is the mass density, $A = bh$ and $I = bh^3/12$ denote the area and area moment of inertia, respectively, E is Young's modulus, G is the shear modulus, $k_s = (5+5\nu)/(6+5\nu)$ is the shear correction factor, where ν is Poisson's ratio. Anchor loss is also relevant for the clamped-clamped beam resonator and will be included later. Additionally, the model includes thermoelastic damping by a coupling of (1)-(2) to the thermal field $\theta(x, z, t)$ through terms:

$$M_T = E\alpha b \int_{-h/2}^{h/2} z\theta dz, \quad N_T = E\alpha b \int_{-h/2}^{h/2} \theta dz, \quad (3)$$

where α is the thermal expansion coefficient, z is the coordinate along the beam width and is θ the temperature profile. The heat equation, describing the thermal field in the beam is based on [5] and extends results from [6]:

$$\rho c_p \frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial z^2} - T_0 \alpha E \frac{\partial}{\partial t} \left[\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + z \frac{\partial \varphi}{\partial x} \right]. \quad (4)$$

Here c_p denotes the heat capacity per unit volume, k is the thermal conductivity and T_0 is the equilibrium temperature. Furthermore, the distributed electrostatic forcing q_e in (1), acting on the beam, includes first-order fringing field correction [7] and is given by:

$$q_e = \frac{\varepsilon_0 b V_1^2(t)}{2(d_1 - w)^2} \left(1 + 0.65 \frac{d_1 - w}{b} \right) - \frac{\varepsilon_0 b V_2^2}{2(d_2 - w)^2} \left(1 + 0.65 \frac{d_2 - w}{b} \right). \quad (5)$$

Actuation voltages are given by $V_1 = V_{dc} + V_{ac} \sin(2\pi f t)$ and $V_2 = V_{dc}$, see Fig. 1. Expression (5) is in line with recently reported work on an electrostatically actuated microbeam [8]. Boundary conditions for (1)-(2) and (4) correspond to clamped edge and insulated boundary conditions, respectively, and are given by:

$$w = 0, \quad \varphi = 0 \quad \text{at } x = 0, l, \quad \text{and} \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{at } z = \pm \frac{h}{2}. \quad (6)$$

A reduced-order model is derived by applying Galerkin discretization to (1)-(6), see [9]. In this approach, the solution to the partial differential equations (1)-(2) and (4) can be approximated by a finite number of spatial basis functions with time-varying coefficients. Undamped, uncoupled modes are used as basis functions for this approach. An accurate approximation has been obtained by using only a single basis function for each field variable w , φ and θ . A single mode discretization is found to be sufficient for obtaining accurate results. Furthermore, anchor loss is included in the reduced-order model by means of a Q -factor. The value for the Q -factor is determined from experiments. After the procedure of Galerkin discretization, a set of ordinary differential equations is obtained, which is solved by numerical techniques, using AUTO97 [10], in order to calculate steady-state periodic solutions (resonator vibrations) for various excitation frequencies f .

4. Results

Model parameters that are used for simulations with the model are listed in Table 1. For a range of excitation values V_{dc} and V_{ac} , both simulations and experiments have been performed, in which characteristic nonlinear dynamic behaviour called frequency hysteresis [11] has been observed. In the experiment, slow frequency sweeps are performed. In Fig. 2, the measured and simulated output V_{out} is depicted versus the excitation frequency. The measured response is observed to jump between high-amplitude and low-amplitude solutions, see Fig. 2(a). In steady-state simulations, this translates to coexisting stable and unstable periodic solutions at a single frequency.

Table 1. Parameters for the model of the single-crystal clamped-clamped beam MEMS resonator.

Parameter	Value	Unit	Parameter	Value	Unit
ρ	2329	kg m ⁻³	T_0	300	K
E	130.02	GPa	h	4.0	μm
ν	0.2785	—	b	1.4	μm
k	156	W m ⁻¹ K ⁻¹	l	47.75	μm
c_p	716	J kg ⁻¹ K ⁻¹	d_1, d_2	342	nm
α	2.616·10 ⁻⁶	K ⁻¹			

Experimentally observed jumps in Fig. 2(a) between stable solution branches are characterized by cyclic fold bifurcations, indicated by CF in Fig. 2(b), see [11]. A cyclic fold bifurcation marks a transition from a stable to an unstable periodic solution, or vice versa. The resonance peak may bend to higher or lower frequencies, which is called hardening or softening nonlinear behavior, respectively. It can be observed that for increasing V_{dc} -values, the response of the resonator undergoes a transition from hardening to softening. At $V_{dc} = 20$ V, the response is without frequency hysteresis. Taking into account the effect of thermal noise, simulations match experiments quite well.

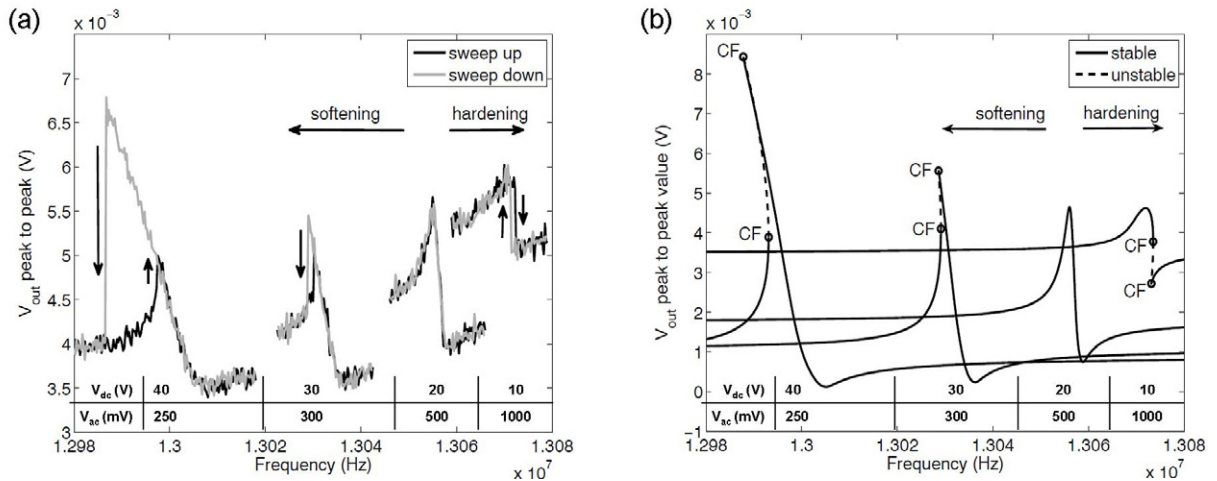


Fig. 2. Transitioning from hardening to softening nonlinear behavior for various excitation parameters: (a) experiments; (b) simulations.

5. Conclusions

From the good agreement between the simulations and experiments for a clamped-clamped beam MEMS resonator, confidence is gained in the first-principles based modeling approach proposed in this paper. The main advantage of the proposed approach is that the model is in parameterized form, containing actual physical parameters, instead of a lumped or heuristic description. In this way, contributions of various different effects, like midplane stretching, fringing fields and thermoelastic damping, can be assessed in a straightforward way. The modeling approach enables parameter studies and design optimization with respect to nonlinear dynamic behavior.

References

1. Nguyen CT-C, MEMS Technology for Timing and Frequency Control, *IEEE Trans Ultrason Ferr* 2007;**54**(2):251-70.
2. Kaajakari V, Mattila T, Oja A, Seppä H, Nonlinear Limits for Single-Crystal Silicon Micro-resonators, *J Microelectromech Syst* 2004;**13**:715-724.
3. Mestrom RMC, Fey RHB, Beek JTM van, Phan KL, Nijmeijer H, Modelling the dynamics of a MEMS resonator: Simulations and experiments, *Sens Actuat A* 2008;**142**:306-315.
4. Shao LC, Wong CL, Palaniapan M, Study of the nonlinearities in micromechanical clamped-clamped beam resonators using stroboscopic SEM, *J Micromech Microeng* 2008;**18**:085019(7pp).
5. Nowinsky JL. *Theory of thermoelasticity with applications*. Slijthof & Noordhoff International Publishers, the Netherlands; 1978.
6. Lifshitz R, Roukes ML, Thermoelastic damping in micro- and nanomechanical systems, *Physical Review B* 2000;**61**(8):5600–5609.
7. Osterberg PM, Senturia SD, M-Test: A Test Chip for MEMS Material Property Measurement Using Electrostatically Actuated Test Structures, *J Microelectromech Syst* 1997;**6**(2):107-118.
8. Krylov S, Harari I, Cohen Y, Stabilization of Electrostatically Actuated Microstructures Using Parametric Excitation, *J Micromech Microeng* 2005;**15**(6):1188–1204.
9. Meirovitch L, *Fundamentals of Vibrations*. McGraw-Hill, London; 2001.
10. Doedel E, Champneys AR, Fairgrieve TF, Kuznetsov, YA, Sandstede B, WANG X, *AUTO97: Continuation and Bifurcation Software for Ordinary Differential Equations (with HomCont)*, Technical Report. Concordia University; 1998.
11. Thomsen JJ, *Vibrations and Stability; Advanced Theory, Analysis and Tools*. Springer Verlag, Berlin, 2nd edition; 2003